Deadbeat Control for a Class of Lumped Parameter Systems

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In recent years considerable attention has been directed toward the time-optimal control of multivariate systems. The general time-optimal control problem is structured such that the control action is bounded, but very few authors have considered the problem with the addition of state constraints.

This note is concerned with a specific type of state constrained time-optimal problem called the *deadbeat* or *critically damped* problem. The objective is to bring a system from some initial state to some desired final state in a minimum amount of time without any of the states crossing over the boundary defined by the desired final state. Lesser and Lapidus (1966) have presented a technique for the solution of the discretized time optimal problem that can handle state constraints by incorporating them into a linear programming algorithm. In the process, however, the dimensionality of the linear programming problem can become large. The deadbeat problem can be handled by the linear programming technique, but in addition to the increase in dimensionality, another difficulty can be encountered. Koppel (1968) shows that when a continuous time model is discretized (as is commonly done in digital control systems) the possibility of hidden oscillations is introduced. The consequence of this is that any algorithm that produces a discretized control policy may in reality produce one that causes the states to violate the desired constraints between the sampling instants. The algorithm to be presented in the next section provides an alternate technique for the calculation of deadbeat control policies for a special class of lumped parameter systems, which does not require an exceptional amount of computer storage and is implemented in the continuous time domain.

DEVELOPMENT OF THE ALGORITHM

The algorithm to be presented is applicable to continuous linear systems which have the following structure. First, each control action must have a direct effect on only the time derivative of one variable and each control action must be bounded. Secondly, the transition matrix for the system must be structured such that a state cannot change sign if adjacent states are of the same sign. Lastly, after the desired state has been translated to the origin, the initial states must have the same sign.

Most engineering systems satisfy the second requirement. When a system of engineering interest is moving from one equilibrium state to another the third requirement is also generally satisfied. The most limiting restriction to the application of the algorithm is the first requirement. A physical interpretation of that requirement is that the control action forcing a system must influence only discrete, individual variables. This is encountered in systems where concentration or temperature are the control variables. The 6-plate absorber described by Lapidus and Luus (1967) with inlet concentration control is an example. A lumped model description of a slab being heated at its boundary is also an example. Lastly, any nth order system that has been transformed into n first order systems is another suitable example.

A system which does not satisfy the first requirement is the liquid-liquid countercurrent extraction train also presented by Lapidus and Luus (1967). Since the flow rate of raffinate solvent is being controlled, all the state variables are directly affected by the control action. Another system which does not meet the first requirement is the linearized model of a nonisothermal stirred tank reactor with inlet flow rate and coolant flow rate control presented by Denn (1969). In this case the inlet flow rate directly affects the temperature and composition in the tank. If, however, the inlet flow rate is held constant the equations reduce to a system that does satisfy the first requirement.

An algorithm to calculate the time-optimal deadbeat control policy for a system meeting these requirements is as follows.

1. Apply the largest amount of control action possible, within the constraints, that forces the states directly influenced by the control action to the origin. For convenience, these states will be referred to as key states.
2. As each key hits the origin, the control action is varied such that these keys remain at the origin. This is accomplished by forcing the time derivative of each key to be zero.
3. Continue to keep the keys at the origin until all the states are within the desired tolerance of the origin.

This algorithm is illustrated by the following example.

EXAMPLE

Consider the 6-plate absorber system described by Lapidus and Luus (1967). This system is defined by

$$\dot{\underline{x}} = \underline{A}\underline{x} + \underline{B}\underline{u} \tag{1}$$

where

$$\underline{A} = \begin{bmatrix} -1.173113 & 0.634115 & 0 & \cdots & & 0 \\ 0.538998 & -1.173113 & 0.634115 & 0 & & \cdot \\ 0 & 0.538998 & -1.173113 & 0.634115 & & \cdot \\ 0 & 0 & 0.538998 & -1.173113 & 0.634115 & 0 \\ 0 & 0 & & 0.538998 & -1.173113 & 0.634115 \\ 0 & 0 & \cdots & 0 & 0.538998 & -1.173113 \end{bmatrix}$$

and

$$\underline{B} = \begin{bmatrix} 0.538998 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0.634115 \end{bmatrix}.$$

The initial value of the state vector $\underline{x}$ is

$$\underline{x}(0) = \begin{bmatrix} -0.0306632 \\ -0.0567271 \\ -0.0788812 \\ -0.0977124 \\ -0.1137188 \\ -0.1273242 \end{bmatrix}$$

and the control vector is bounded such that

$$0 \leq u_1 \leq 1.0, \quad -0.4167 \leq u_2 \leq 0.972.$$

This system meets the requirements of the algorithms since

1. u_1 affects only $\dot{x}_1$ and u_2 affects only $\dot{x}_6$,
2. the normalized initial states are of the same sign,
3. and the x_i's between x_1 and x_6 cannot change sign as long as x_1 and x_6 do not change sign.

The application of the proposed algorithm is as follows. First at $t = 0$, the constrained value of the controls $u_1 = 1.0$ and $u_2 = 0.972$ are applied. At $t = 0.0586$, $x_1 = 0.000$ so u_1 is found from

$$u_1 = -(1/B_{1,1})\ [\underline{A}_1]\ \underline{x}$$

where $[A_i]$ represents the ith row of the $\underline{A}$ matrix. u_2 is kept at 0.972 until $t = 0.2025$. At that point $x_6 = 0.000$ and u_2 is found from

$$u_2 = -(1/B_{6,2})\ [\underline{A}_6]\ \underline{x}$$

From hence, u_1 and u_2 are calculated in a closed loop fashion until all the x_i are within a desired tolerance of the origin.

SUMMARY

An algorithm for generating the time-optimal deadbeat control policies for a class of lumped parameter systems has been presented and illustrated. The algorithm avoids the possibility of hidden oscillation, possible with other techniques proposed for solving this problem, because it is a continuous time algorithm. In addition, a portion of the algorithm is closed loop in nature, and only a small amount of computer storage is required to implement the solution.

LITERATURE CITED

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Viscosity of Saturated Nonpolar Liquids at Elevated Pressures

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Numerous theoretical and experimental studies have been conducted on the viscosity of saturated liquids. Reid and Sherwood (1966) combined relationships of Thodos et al. (1962) for the dimensionless residual viscosity of dense gases and liquids as functions of reduced density with the equations of Stiel and Thodos (1961) for dilute nonpolar gases and the density correlation of Lydersen et al. (1955). The resulting procedure for the viscosity of saturated nonpolar liquids, valid between the normal boiling point and the critical point, is of the following functional form:

$$\mu\xi = f(T_R, z_c) \qquad (1)$$

where $\mu\xi R^{1/6}$ is dimensionless. In the present study, available viscosity data for a number of nonpolar liquids have been utilized to relate the dimensionless viscosity directly to reduced temperature and the equivalent third parameter for nonpolar fluids, the acentric factor, for reduced temperatures from 0.76 to 1.00.

In Figure 1 values of ln $\mu\xi$ are plotted against $1/T_R$ for *n*-butane ($\omega = 0.201$) and iso-butane ($\omega = 0.192$). It can be seen that at low temperatures to approximately $T_R = 0.7$, the saturated liquid viscosity follows the relationship (Reid and Sherwood, 1966)

$$\mu\xi = A\ e^{B/T_R} \qquad (2)$$

where A and B are constants for each substance which are determined primarily by the structure of the molecules. For isomers of *n*-pentane, *n*-hexane, and *n*-heptane, the slope of ln $\mu\xi$ versus $1/T_R$ also increases as the molecule becomes more spherical. There is no simple method to

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